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表层 Ekman 剪切流场中的瞬时点源扩散*

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海洋和大气中的扩散现象，由于乱流（湍流）的存在，致使分子扩散方程不能很好地进行描述。本世纪初人们开始对湍流扩散进行了较多的研究，Taylor（1922）提出了用 Lagrange 相关函数研究湍流扩散的方法；Prandtl（1925）提出了混合长度概念；Richardson（1926）提出了相邻扩散概念和相邻扩散方程。到五十年代，Taylor（1953），Corrsin（1953）和 Bowles 等（1958）指出了流速切变（也称剪流）对扩散的重要作用，后经 Okubo[3-5] 的研究证明，湍流和流速切变同湍流间的相互作用是扩散的两个重要机制。这是扩散研究中的进展之一。近年来，也有人对扩散问题进行过数值计算。在进行理论研究的同时，人们已做了许多扩散实验，如染色扩散实验和扩散的物理模拟等。实验揭示了一些新的现象。关于扩散实验已揭露的现象，目前有的已得到了说明，有的尚只有某些推测。

本文拟对表层 Ekman 漂流场中的瞬时点源的被动扩散问题进行一些初步分析，并对染色实验所揭露的现象作一些说明。

一、几 点 现 象

五十年代以来，人们作了许多染色扩散实验，Okubo[6] 在报告中提到了这些实验所揭示的现象，现将其集中起来，简述如下：

1. 许多关于瞬时点源染料团扩散实验的航空照片表明，扩散着的染料团或多或少是被拉长了的。

2. 在最初两个小时左右，染料团大致是在局地风的方向被拉长的。

3. 扩散着的染料团的前部（头部，即下风方向一端）比其后部（尾部，即上风方向一端）有较高的染料浓度。

4. 染料团头部的染色最浓处并不在染料团的最前沿，而在头部的离开前沿稍往后一点的地方。

5. 观测表明，染料团的前部（头部）位于海表面，或近海表面处；尾部位于海表面以下几米深的水层中。从前沿至尾部，染料团所处的深度是逐渐增加的。

6. 外海三维的染色扩散实验表明，其中心最大浓度与时间 $t^{-1.5}$ 和 $t^{-2.5}$ 成比例；而近岸二维的染色扩散实验表明，其中心最大浓度与时间 $t^{-1.0}$ 和 $t^{-2.0}$ 成比例。

7. 当从染料团的头部向尾部看去时，可见到尾部的弯曲：在北半球是顺时针方向的

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弯曲;也有不多的实验表明,在南半球是逆时针方向的弯曲。

8. 连续点源扩散,染色的浓度与离源的距离成反比。

二、方程的求解

一般的平流扩散方程,在 x 轴向东、y 轴向北、z 轴向下、且坐标原点在平均海平面上的直角坐标系中,有如下形式:

$$\frac{\partial \varphi}{\partial t}+u\frac{\partial \varphi}{\partial x}+v\frac{\partial \varphi}{\partial y}+w\frac{\partial \varphi}{\partial z}=\frac{\partial}{\partial x}\left(K\frac{\partial \varphi}{\partial x}\right)+\frac{\partial}{\partial y}\left(M\frac{\partial \varphi}{\partial y}\right)+\frac{\partial}{\partial z}\left(N\frac{\partial \varphi}{\partial z}\right)+R, \tag{1}$$

式中 φ 为扩散物质的浓度, u、v、w 分别为 x、y、z 方向的流速分量, K、M、N 分别为 x、y、z 方向的湍流扩散系数,R 为扩散物质的连续的源或汇。

假定扩散物质的密度与周围介质(海水)的密度相同,并假定扩散物质的引入对周围介质的流场无所影响或影响可略;在我们研究的区域内没有连续的源或汇,即 $R=0$; 湍流扩散系数 K、M、N 为常数。

周围介质的流场为无界的无限深海 Ekman 漂流,当均匀风沿 y 轴方向吹时:

$$\left.\begin{aligned} u&=Ve^{-\frac{\pi}{D}z}\cos\left(\frac{\pi}{4}-\frac{\pi}{D}z\right),\\ v&=Ve^{-\frac{\pi}{D}z}\sin\left(\frac{\pi}{4}-\frac{\pi}{D}z\right),\\ w&\approx 0,\end{aligned}\right\} \tag{2}$$

其中 V 为表面流速模,是常数,D 为 Ekman 摩擦深度。

此时我们的方程变为:

$$\begin{aligned}\frac{\partial \varphi}{\partial t}&+Ve^{-\frac{\pi}{D}z}\cos\left(\frac{\pi}{4}-\frac{\pi}{D}z\right)\frac{\partial \varphi}{\partial x}+Ve^{-\frac{\pi}{D}z}\sin\left(\frac{\pi}{4}-\frac{\pi}{D}z\right)\frac{\partial \varphi}{\partial y}\\ &=K\frac{\partial^2 \varphi}{\partial x^2}+M\frac{\partial^2 \varphi}{\partial y^2}+N\frac{\partial^2 \varphi}{\partial z^2}。\end{aligned} \tag{3}$$

由于染色扩散实验多数是在海面投入染料的,所以揭露的现象也主要是表层或上层扩散的特点。因为我们讨论的是表层的扩散问题,深度不大, z/D 是小量,所以方程可进行一些简化。

将 Ekman 流速 u 和 v 在 $z=0$ (海表面)附近展开为泰勒级数,

$$\left.\begin{aligned} u&=Ve^{-\frac{\pi}{D}z}\cos\left(\frac{\pi}{4}-\frac{\pi}{D}z\right)=\frac{V}{\sqrt{2}}\left(1-\frac{\pi^2}{D^2}z^2+\cdots\cdots\right),\\ v&=Ve^{-\frac{\pi}{D}z}\sin\left(\frac{\pi}{4}-\frac{\pi}{D}z\right)=\frac{V}{\sqrt{2}}\left(1-2\frac{\pi}{D}z+\frac{\pi^2}{D^2}z^2+\cdots\cdots\right),\end{aligned}\right\} \tag{4}$$

取一阶近似,并令 $U=\dfrac{V}{\sqrt{2}}$, 则:

$$\left.\begin{aligned} u &\approx U, \\ v &\approx U\left(1-2\frac{\pi}{D}z\right)。\end{aligned}\right\} \tag{5}$$

将(5)式中的 u 和 v 的近似式代入(3)式，则：

$$\frac{\partial\varphi}{\partial t}+U\frac{\partial\varphi}{\partial x}+U\left(1-2\frac{\pi}{D}z\right)\frac{\partial\varphi}{\partial y}=K\frac{\partial^2\varphi}{\partial x^2}+M\frac{\partial^2\varphi}{\partial y^2}+N\frac{\partial^2\varphi}{\partial z^2}, \tag{6}$$

初始条件为

$$\varphi|_{t=0}=2C\delta(x)\,\delta(y)\,\delta(z) \tag{7}$$

式中 C 为常数，$\delta(x)$，$\delta(y)$ 和 $\delta(z)$ 为 δ-函数。

假定 φ 是保守的，则 φ 应满足条件：

$$\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\int_{0}^{\infty}\varphi\,dz=C \tag{8}$$

作自变量代换，令 $\xi=x-Ut$，$\eta=y-U\left(1-2\frac{\pi}{D}z\right)t$，$z=z$，$t=t$，则(6)式变为：

$$\frac{\partial\varphi}{\partial t}=K\frac{\partial^2\varphi}{\partial\xi^2}+M\frac{\partial^2\varphi}{\partial\eta^2}+N\frac{\partial^2\varphi}{\partial z^2}+4\frac{\pi}{D}UtN\frac{\partial^2\varphi}{\partial\eta\partial z}+4\frac{\pi^2}{D^2}U^2t^2N\frac{\partial^2\varphi}{\partial\eta^2}。 \tag{9}$$

现将半无限空间（$z\geqslant 0$）上的问题，形式地在全空间上求解，使所得结果在 $z\geqslant 0$ 的半空间上恰是我们的问题的解。为此作函数 ϕ，使得当 $z\geqslant 0$ 时，有 $\phi\equiv\varphi$，则问题变为：

$$\frac{\partial\phi}{\partial t}=K\frac{\partial^2\phi}{\partial\xi^2}+M\frac{\partial^2\phi}{\partial\eta^2}+N\frac{\partial^2\phi}{\partial z^2}+4\frac{\pi}{D}UtN\frac{\partial^2\phi}{\partial\eta\partial z}+4\frac{\pi^2}{D^2}U^2t^2N\frac{\partial^2\phi}{\partial\eta^2} \tag{10}$$

$$\phi|_{t=0}=2C\delta(\xi)\delta(\eta)\delta(z), \tag{11}$$

$$\iiint_{-\infty}^{\infty}\phi\,d\xi\,d\eta\,dz=2C。 \tag{12}$$

作 Fourier 变换：

$$\tilde{\phi}(\alpha,\beta,\gamma)=\iiint_{-\infty}^{\infty}\phi(\xi,\eta,z)\,e^{-i(\alpha\xi+\beta\eta+\gamma z)}\,d\xi\,d\eta\,dz, \tag{13}$$

则(10)式变为：

$$\frac{\partial\tilde{\phi}}{\partial t}=\left(-K\alpha^2-M\beta^2-N\gamma^2-4\frac{\pi}{D}UtN\beta\gamma-4\frac{\pi^2}{D^2}U^2t^2N\beta^2\right)\tilde{\phi} \tag{14}$$

将(14)式对 t 积分得：

$$\tilde{\phi}=Ae^{-K\alpha^2t-M\beta^2t-N\gamma^2t-2\frac{\pi}{D}Ut^2N\beta\gamma-\frac{4}{3}\frac{\pi^2}{D^2}U^2t^3N\beta^2} \tag{15}$$

式中 A 为积分常数，据初条件(11)式得 $A=2C$。将(15)式作 Fourier 反变换：

$$\phi(\xi,\eta,z)=\frac{1}{8\pi^3}\iiint_{-\infty}^{\infty}\tilde{\phi}e^{i(\alpha\xi+\beta\eta+\gamma z)}\,d\alpha\,d\beta\,d\gamma, \tag{16}$$

则有：

$$\phi=2CJ_1J_2, \tag{17}$$

$$J_1=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-K\alpha^2t+i\alpha\xi}\,d\alpha=\frac{1}{2\sqrt{K\pi t}}\,e^{-\frac{\xi^2}{4Kt}}, \tag{18}$$

$$J_2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-\left(Mt + \frac{4}{3}\frac{\pi^2}{D^2}U^2t^3N\right)\beta^2 + i\eta\beta} J_3 d\beta, \tag{19}$$

$$J_3 = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-Nt\gamma^2 - 2\frac{\pi}{D}Ut^2N\beta\gamma + iz\gamma} d\gamma = \frac{1}{2\sqrt{N\pi t}} e^{\frac{\left(2\frac{\pi}{D}Ut^2N\beta - iz\right)^2}{4Nt}}$$

$$= \frac{1}{2\sqrt{N\pi t}} e^{+\frac{z^2}{4Nt}} e^{\frac{\pi^2}{D^2}U^2t^3N\beta^2} e^{-i\frac{\pi}{D}Utz\beta}。 \tag{20}$$

将(20)式的 J_3 代入(19)式的 J_2 并积分得:

$$J_2 = \frac{1}{4\pi\sqrt{N\pi t}}\int_{-\infty}^{\infty} e^{-\left(Mt+\frac{4}{3}\frac{\pi^2}{D^2}U^2t^3N\right)\beta^2+i\eta\beta} e^{-\frac{z^2}{4Nt} + \frac{\pi^2}{D^2}U^2t^3N\beta^2 - i\frac{\pi}{D}Utz\beta} d\beta$$

$$= \frac{1}{4(N\pi t)^{1/2}(M\pi t)^{1/2}\left(1 + \frac{1}{3}\frac{\pi^2}{D^2}U^2t^2\frac{N}{M}\right)^{1/2}} e^{-\frac{z^2}{4Nt}} e^{-\frac{\left(\eta - \frac{\pi}{D}Utz\right)^2}{4Mt\left(1+\frac{1}{3}\frac{\pi^2}{D^2}U^2t^2\frac{N}{M}\right)}}。 \tag{21}$$

将(18)式的 J_1 和(21)式的 J_2 代入(17)式得:

$$\phi = \frac{2C}{8(\pi t)^{3/2}(KMN)^{1/2}\left(1 + \frac{1}{3}\frac{\pi^2}{D^2}U^2t^2\frac{N}{M}\right)^{1/2}} e^{-\frac{\xi^2}{4Kt}} e^{-\frac{\left(\eta - \frac{\pi}{D}zUt\right)^2}{4Mt\left(1+\frac{1}{3}\frac{\pi^2}{D^2}U^2t^2\frac{N}{M}\right)}} e^{-\frac{z^2}{4Nt}}, \tag{22}$$

还原为原来的变数,则:

$$\varphi = \frac{C}{4(\pi t)^{3/2}(KMN)^{1/2}\left(1 + \frac{1}{3}\frac{\pi^2}{D^2}U^2t^2\frac{N}{M}\right)^{1/2}}$$

$$\cdot \exp\left\{-\frac{(x - Ut)^2}{4Kt} - \frac{\left(y - Ut + \frac{\pi}{D}zUt\right)^2}{4Mt\left(1+\frac{1}{3}\frac{\pi^2}{D^2}U^2t^2\frac{N}{M}\right)} - \frac{z^2}{4Nt}\right\}。 \tag{23}$$

解式(23)经代回原方程验算,满足原方程(6),且满足初始条件 (7),并满足描述保守的扩散物质在海水中的总量守恒的条件(8),所以 φ 的表达式(23)是我们的问题的一个特解。

三、关于解的讨论

解式(23)有如下特征:

1. 染料团中心处的最大浓度,在不同深度上是不同的,它随深度而减小,其表达式为:

$$\varphi_{\max,z} = \frac{C}{4(\pi t)^{3/2}(KMN)^{1/2}\left(1 + \frac{1}{3}\frac{\pi^2}{D^2}U^2t^2\frac{N}{M}\right)^{1/2}} e^{-\frac{z^2}{4Nt}}。 \tag{24}$$

在海表面 ($z = 0$) 处:

$$\varphi_{\max,0} = \frac{C}{4(\pi t)^{3/2}(KMN)^{1/2}\left(1 + \frac{1}{3}\frac{\pi^2}{D^2}U^2t^2\frac{N}{M}\right)^{1/2}}。 \tag{25}$$

由(25)式可见，当 $\frac{1}{3}\frac{\pi^2}{D^2}U^2t^2\frac{N}{M} \ll 1$ 时，即扩散开始不久，

$$\varphi_{\max,0} \approx \frac{C}{4(\pi)^{3/2}(KMN)^{1/2}}t^{-1.5}, \tag{26}$$

当 $\frac{1}{3}\frac{\pi^2}{D^2}U^2t^2\frac{N}{M} \gg 1$ 时，即扩散已进行了一段时间以后，

$$\varphi_{\max,0} \approx \frac{\sqrt{3}\,CD}{4(\pi)^{5/2}(K)^{1/2}UN}t^{-2.5}。 \tag{27}$$

以上的(26)式和(27)式同第一节中所引的观测结果是相符合的。

2. 令解式(23)左端的 φ 为常数 φ_i，将左右两端都取自然对数，则：

$$\frac{(x-Ut)^2}{4Kt}+\frac{\left(y-Ut+\frac{\pi}{D}zUt\right)^2}{4Mt\left(1+\frac{1}{3}\frac{\pi^2}{D^2}U^2t^2\frac{N}{M}\right)}+\frac{z^2}{4Nt}=\ln\frac{\varphi_{\max,0}}{\varphi_i}。 \tag{28}$$

由(28)式可见，当 $K\approx M$ 时，在扩散开始阶段，各水平面上 φ_i 的等值线略呈圆形；随着时间的增长，沿着风吹的方向(在我们的表达式中是 y 轴方向)逐渐被拉长，变成椭圆形。

3. 染料团的最大浓度点，在各个时刻的位置的坐标为：

$$P(x,y,z)=P\left(Ut,\ Ut-U\frac{\pi}{D}zt,\ z\right)。 \tag{29}$$

可见在海表面及其附近，最大浓度点的 x 坐标(垂直于风向)与深度无关，而 y 坐标(平行于风向)随深度减小。这表示染料团的头部在下风端，且处于海表面，而从头至尾最大浓度点逐渐下沉，深度逐渐增加，其最大浓度点的联线与风向平行。

4. 由于染料团最大浓度点的浓度数值随深度增加按 $e^{-\frac{z^2}{4Nt}}$ 的方式而减小，浓度数值最大者在海表面，越往深处中心浓度越小，其移动速度也是在海表面最大，越往深处越小，染料团在海面的一层跑在整个染料团的前部，故显出了染料团前部(头部)染色浓度较高的特征。

5. 从海面看，染料团的前部浓度较高(前面已作说明)，但由观测发现最高浓度值并不在染料团的最前沿，而是在离开最前沿稍微往后一点的地方。为何出现这种现象，Okubo 曾指出是因为存在纵向扩散的缘故。由上面的解式来分析，这一现象的出现，除 Okubo 指出的原因外，可能还由于上、下层染料团中心最大浓度点的位置不在一条垂直线上，而是下层比相邻的上层相对地落后，于是表面层最大浓度处的后一部分与下一层最大浓度处的前部上下叠置，形成了在离前沿稍微往后一点的地方比染料团最前沿染色更浓的表观。

6. 也应指出，由于我们对流场取了一阶近似表示，故所说明的只是在表层和靠近表层的现象，至于染料团尾部的弯曲和深层扩散的情况本文没有讨论，也许对流场取二阶或更高阶近似，或更准确的表达式，染料团尾部的弯曲现象是可以得到说明的，不过那要用到更复杂的数学。关于连续源的扩散本文也未涉及，待以后再作探讨。

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SHEAR DIFFUSION FROM AN INSTANTANEOUS POINT-SOURCE IN EKMAN DRIFT CURRENT FIELD OF SEA SURFACE LAYER*

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Abstract

Since the early fifties, a large number of dye release experiments have been made on the sea, and many new diffusion phenomena revealed. Of these new phenomena some characteristics have been explained, but most of them have not yet. In this article, a preliminary account was given to the passive shear diffusion from an instantaneous point-source in the surface Ekman drift current field of a vast and deep sea.

To begin with the general advection diffusion equation, suppose the diffusion is passive, using a left-handed Cartesian coordinate system, the origin of which is on the mean surface, the positive direction of Z downwards, and the local wind is uniform, blowing along the Y-axis, we may define our problem as follows:

$$\left.\begin{aligned} &\frac{\partial\varphi}{\partial t}+Ve^{-\frac{\pi}{D}z}\cos\left(\frac{\pi}{4}-\frac{\pi}{D}z\right)\frac{\partial\varphi}{\partial x}+Ve^{-\frac{\pi}{D}z}\sin\left(\frac{\pi}{4}-\frac{\pi}{D}z\right)\frac{\partial\varphi}{\partial y}\\ &\qquad=K\frac{\partial^2\varphi}{\partial x^2}+M\frac{\partial^2\varphi}{\partial y}+N\frac{\partial^2\varphi}{\partial z^2},\\ &\varphi|_{t=0}=2C\delta(x)\delta(y)\delta(z),\\ &\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\int_0^{\infty}\varphi dz=C, \end{aligned}\right\} \quad (1)$$

where φ is the concentration of the material released; D the Ekman friction depth; C,

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the total of the material released. K, M, N, and V are assumed constant.

Since Z/D is small in surface layer, we may expand the velocity in Taylor series about Z = 0, take the first order approximation, $u \approx \frac{V}{\sqrt{2}} = U$, $v \approx U\left(1 - 2\frac{\pi}{D}z\right)$, of the series in place of the velocity in equation, put $\xi = x - Ut$, $\eta = y - U\left(1 - 2\frac{\pi}{D}z\right)t$, $Z = Z$, $t = t$, and make a function ϕ such that we have $\phi \equiv \varphi$ when $Z \geqslant 0$, then our problem turns to

$$\left.\begin{aligned}
&\frac{\partial \phi}{\partial t} = K\frac{\partial^2 \phi}{\partial \xi^2} + M\frac{\partial^2 \phi}{\partial \eta^2} + N\frac{\partial^2 \phi}{\partial z^2} + 4\frac{\pi}{D}UtN\frac{\partial^2 \phi}{\partial \eta\,\partial z} + 4\frac{\pi^2}{D^2}U^2t^2N\frac{\partial^2 \phi}{\partial \eta^2},\\
&\phi|_{t=0} = 2C\delta(\xi)\delta(\eta)\delta(z),\\
&\iiint_{-\infty}^{\infty} \phi\, d\xi d\eta dz = 2C.
\end{aligned}\right\} \tag{2}$$

By Fourier transform on ξ, η, z,

$$\tilde{\phi} = \iiint_{-\infty}^{\infty} \phi e^{-i(\alpha\xi+\beta\eta+\gamma z)} d\xi d\eta dz$$

we get

$$\frac{\partial \tilde{\phi}}{\partial t} = -\left(K\alpha^2 + M\beta^2 + N\gamma^2 + 4\frac{\pi}{D}UtN\beta\gamma + 4\frac{\pi^2}{D^2}U^2t^2N\beta^2\right)\tilde{\phi}, \tag{3}$$

obviously, a solution of our problem (2) is

$$\phi = \frac{C}{4(\pi t)^{3/2}(KMN)^{1/2}\left(1 + \frac{1}{3}\frac{\pi^2}{D^2}U^2t^2\frac{N}{M}\right)^{1/2}} \times \exp\left\{-\frac{\xi^2}{4Kt} - \frac{\left(\eta - \frac{\pi}{D}zUt\right)^2}{4Mt\left(1 + \frac{1}{3}\frac{\pi^2}{D^2}U^2t^2\frac{N}{M}\right)} - \frac{z^2}{4Nt}\right\}. \tag{4}$$

Back to original variables, we have

$$\varphi = \frac{C}{4(\pi t)^{3/2}(KMN)^{1/2}\left(1 + \frac{1}{3}\frac{\pi^2}{D^2}U^2t^2\frac{N}{M}\right)^{1/2}} \times \exp\left\{-\frac{(x - Ut)^2}{4Kt} - \frac{\left(y - Ut + \frac{\pi}{D}zUt\right)^2}{4Mt\left(1 + \frac{1}{3}\frac{\pi^2}{D^2}U^2t^2\frac{N}{M}\right)} - \frac{z^2}{4Nt}\right\}. \tag{5}$$

The solution (5) has following characteristics:

1, The maximum concentration on different level is not the same. On sea surface, we have

$$\varphi_{\max,0}=\frac{C}{4(\pi)^{3/2}(KMN)^{1/2}}t^{-1.5},\quad \text{when}\quad \frac{1}{3}\frac{\pi^2}{D^2}U^2t^2\frac{N}{M}\ll 1;$$

$$\varphi_{\max,0}=\frac{\sqrt{3}\,CD}{4(\pi)^{5/2}(K)^{1/2}UN}t^{-2.5},\quad \text{when}\quad \frac{1}{3}\frac{\pi^2}{D^2}U^2t^2\frac{N}{M}\gg 1.$$

2, The distribution of the material is elliptical.

3, It is elongated along local wind direction.

4, The location of the maximum concentration at different level is $P(x, y, z)=P\left(Ut,\ Ut-U\frac{\pi}{D}zt,\ z\right)$, therefore the "head" is located on the sea surface and to the down wind end of the material patch, from head to tail the location of the maximum concentration point is down deeper and deeper, the line joining the maximum concentration points is parallel to the local wind direction.

5, The leading part of the patch has higher concentration than the trailing part of it.

6, Observations showed that the maximum concentration is not just on the front line of the patch, but a little behind, Okubo (1971) have pointed out that this front is well defined but it is not inordinately sharp since longitudinal diffusion softened it. From our solution, it can be thought that, besides what Okubo have told, above mentioned phenomenon is also due to that the maximum concentration points at different level are not in a vertical line, but the more deeper the level, the more backward the maximum concentration is. Thus the rear portion of the maximum concentration on upper level overlaps the front portion of the deeper one.

All things above are satisfactory when compared with observations of dye experiments on the sea quoted by Okubo (1971).

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